Electrical Circuit Fundamentals

Before we can begin the discussion about synchronization of the Grid we must acquire knowledge of some of the fundamentals of electrical networks and the relationships of these components to the differential equations which are used to analyze these components. Knowledge of these basic elements will allow us to model transmission line and how the GRID as a whole can be represented mathematically.

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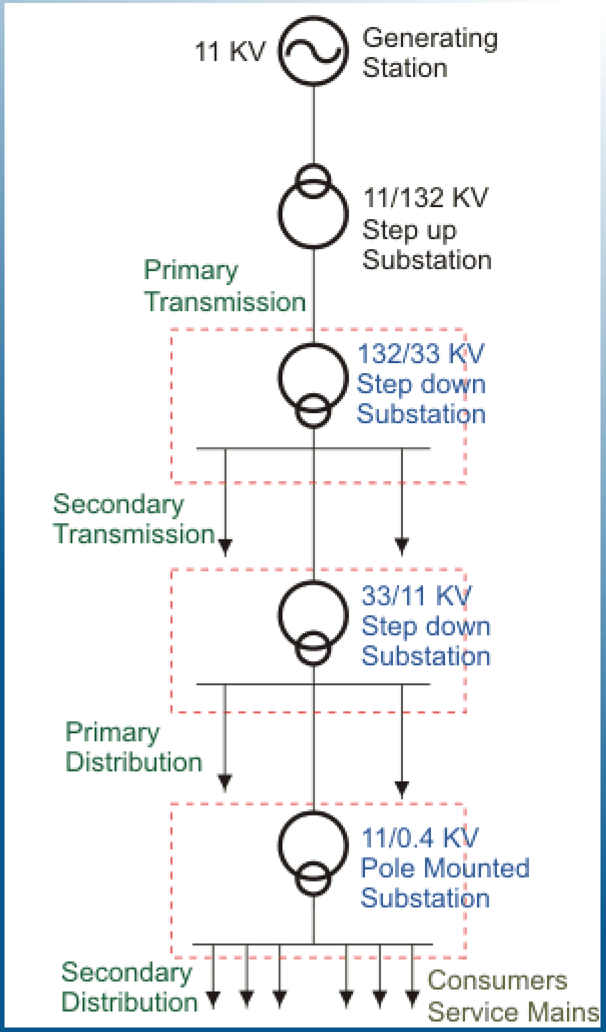
Electrical Circuit Fundamentals

❖ Introduction-One

- ❖ An **electrical circuit or network** is an interconnection of electrical components (e.g., batteries, voltage sources, resistors, inductors, capacitors, switches, transistors) or a model of such an interconnection, consisting of electrical elements (e.g., voltage sources, current sources, resistances, inductances, capacitances).
- ❖ The diagram to the right is a single line diagram of a transmission system which is part of the ERCOT Interconnect.



Electrical Circuit Fundamentals

❖ Introduction-Two

❖ Electron (Unit of Charge)

Electrons play an essential role in numerous physical phenomena, such as electricity, magnetism, chemistry and thermal conductivity. Since an electron has charge, it has a surrounding electric field, and if that electron is in motion then the motion of the electrons creates an electric current.

- Electrons are charge carriers
- Unit of charge is the *Coulomb* (C)
- Current is the rate of flow of charge
- 1C of negative charge = total charge carried by 6.242×10^{18} electrons
- Charge of 1 electron = $1/\ 6.242\times10^{18} = 1.6\times10^{-19}$C
- Charge can either be positive or negative
- Electric current exists when there is a net transfer of charge in a material

Electrical Circuit Fundamentals

- Electrical Components-One
  - Passive Components

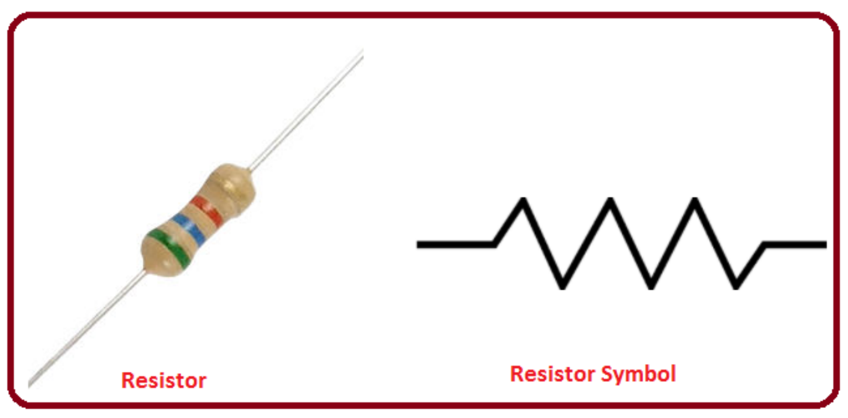
- **Resistors**
  - Pass current in proportion to voltage (Ohm's law) and oppose current.
- **Capacitors**
  - Capacitors store and release electrical charge. They are used for filtering power supply lines, tuning resonant circuits, and for blocking DC voltages while passing AC signals, among numerous other uses.
- **Magnetic (inductive) devices**
  - Electrical components that use magnetism in the storage and release of electrical energy.
- **Transformers**
  - Transformers are used in Transmission Systems for increasing low AC voltages at high current (a step-up transformer) or decreasing high AC voltages at low current (a step-down transformer).

Electrical Circuit Fundamentals

❖ Electrical Components-Two

❖ Resistance

- Resistance is the measure of the extent to which a material interferes with, or resists, the flow of current through it
- A **conductor** has small resistance, an **insulator** has high resistance
- Unit of resistance is the **Ohm** (George Ohm) - *R*
- The symbol of the Ohm is Ω
- For a perfect conductor, its resistance is 0 Ω
- A perfect insulator has a resistance of ∞ Ω
- Conventional current needs a complete path to flow
- There must be a destination that will accept electrons, and there must also be a source of electrons



Electrical Circuit Fundamentals

- ❖ Electrical Components-Two
  - ❖ Conductance

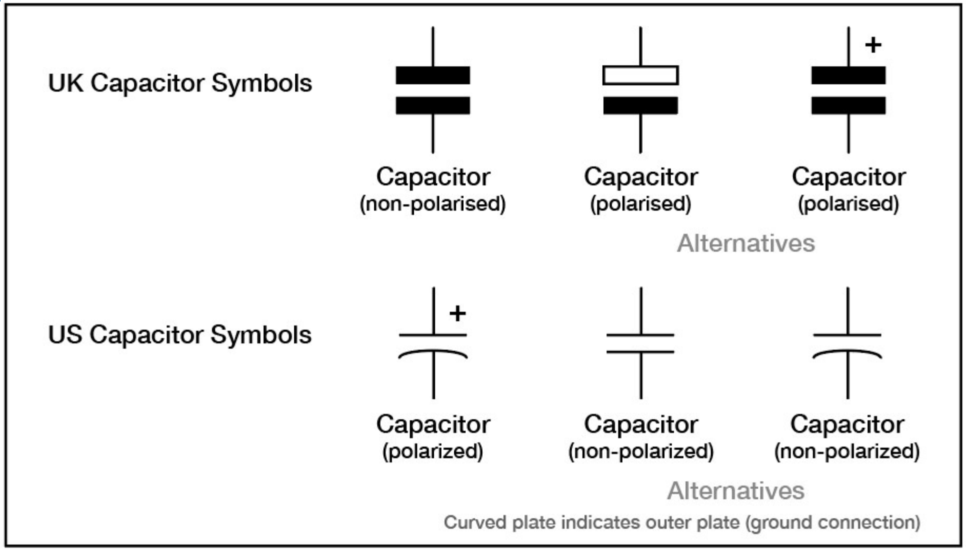
The **electrical resistance** of an object is a measure of its opposition to the flow of electric current. Its reciprocal quantity is **electrical conductance**, measuring the ease with which an electric current passes. Electrical resistance shares some conceptual parallels with mechanical friction. The SI unit of electrical resistance is the ohm (Ω), while electrical conductance is measured in siemens (S) (formerly called "mho"s and then represented by ℧).

- ❖ **Conductance** is the reciprocal of resistance
- ❖ Conductance is the ability of a material to pass electrons
- ❖ The **higher** the resistance, the **lower** the conductance
- ❖ Symbol of Conductance is **G**
- ❖ Unit of conductance is the **Siemen** (**S**)
- ❖ $G = 1/R$ Siemens

Electrical Circuit Fundamentals

- Electrical Components-Three
  - Reactance-One
    - Capacitance

A **capacitor** is a device that stores electrical energy in an electric field. It is a passive electronic component with two terminals.
The effect of a capacitor is known as capacitance. While some capacitance exists between any two electrical conductors in proximity in a circuit, a capacitor is a component designed to add capacitance to a circuit. The capacitor was originally known as a **condenser.**



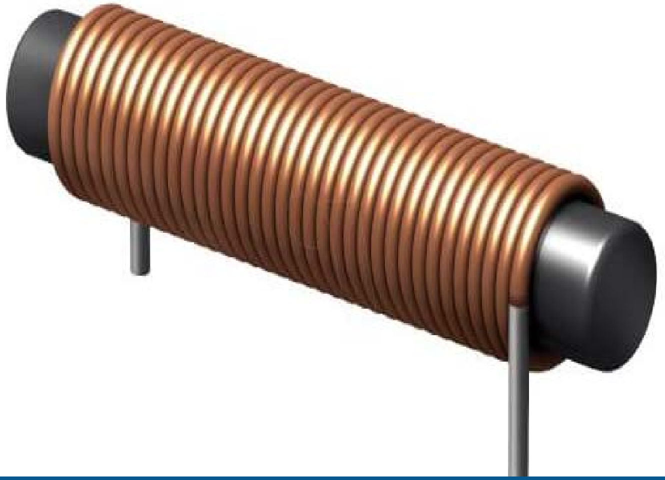
Electrical Circuit Fundamentals

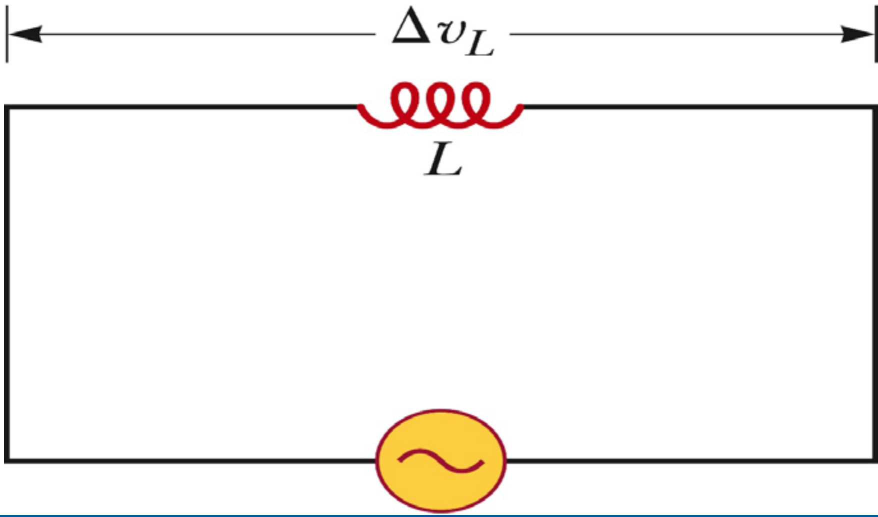
❖ Electrical Components-Four

❖ Reactance-Two

❖ Inductance

- ❖ An **inductor**, also called a **coil**, **choke**, or **reactor**, is a passive two-terminal electrical component that stores energy in a magnetic field when electric current flows through it. An inductor typically consists of an insulated wire wound into a coil.
- ❖ When the current flowing through the coil changes, the time-varying magnetic field induces an electromotive force (*e.m.f.*) (voltage) in the conductor, described by Faraday's law of induction.





Electrical Circuit Fundamentals

❖ Electrical Components-Five

❖ Admittance

Admittance is a measure of how easily a circuit or device will allow a current to flow. It is defined as the reciprocal of impedance, analogous to how conductance & resistance are defined. The SI unit of admittance is the siemens (symbol S); the older, synonymous unit is mho, and its symbol is ℧ (an upside-down uppercase omega Ω)

- Admittance, just like impedance, is a complex number, made up of a real part (the conductance, G), and an imaginary part (the susceptance, B), thus:
- $Y = G + jB$
- where G (conductance) and B (susceptance)

Electrical Circuit Fundamentals

- Electrical Components-Six
  - Transformers-One

- Transformers are electrical devices used to convert or "transform" AC voltage from one level to another. (high to low or low to high)
- Input and output are AC
- They do this by the principle of electromagnetic induction
- Transformers are the second most important machines on the grid after the generation machinery.

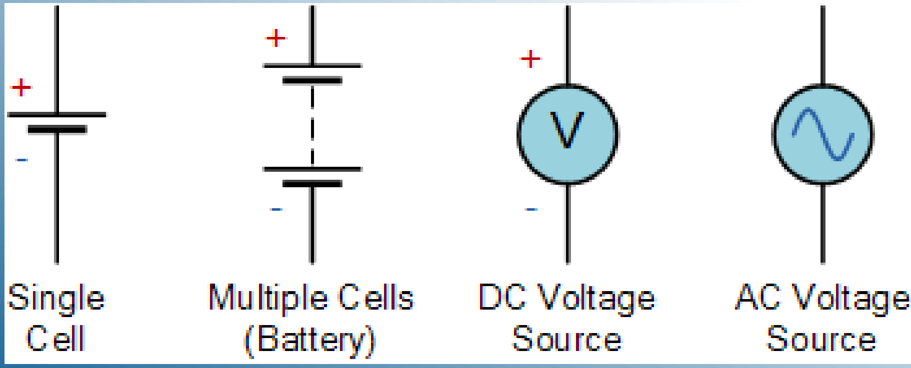
Electrical Circuit Fundamentals

- Electrical Components-Seven
  - Transformers-Two

Electrical Circuit Fundamentals

❖ Electrical Components-Eight

❖ Voltage Source-One



- ❖ To establish a flow of charge through a conductor, we need to exert a force on the electrons that carry the charge
- ❖ This is called **electromotive force (emf)**
- ❖ Unit of EMF is the **volt**
- ❖ The greater the voltage of a source of EMF, the greater the current it can produce
- ❖ EMF is also called **electric potential**, which is the same as talking about the ability (potential) of a voltage source to produce current
- ❖ We say E volts *across* the voltage source or component

Electrical Circuit Fundamentals

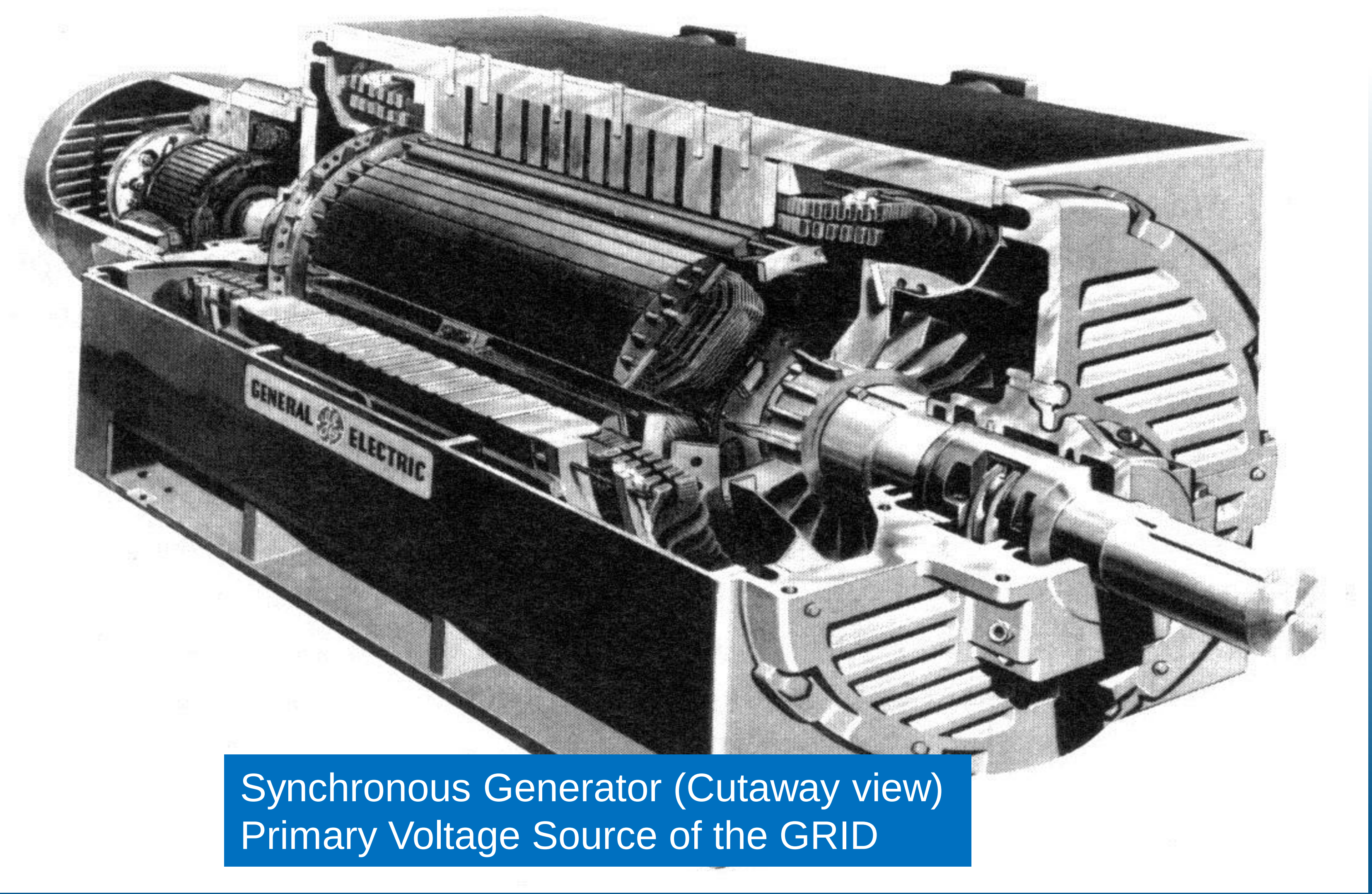
❖ Electrical Components-Nine

❖ Voltage Source-Two

- **Ideal voltage source** – one that maintains a constant terminal voltage independent of the current drawn from it
- **But** all real voltage sources have an internal resistance that causes the terminal voltage to drop if the current is made large, i.e. if a small value of resistance is connected across the terminals
- However, for ease in practical analysis, it is convenient to assume that a voltage source is ideal, i.e. we can neglect its **internal** resistance
- A synchronous generator is often depicted as an Ideal Voltage Source.

Electrical Circuit Fundamentals

- Electrical Components-Ten
  - Voltage Source-Three



Synchronous Generator (Cutaway view)
Primary Voltage Source of the GRID

Electrical Circuit Fundamentals

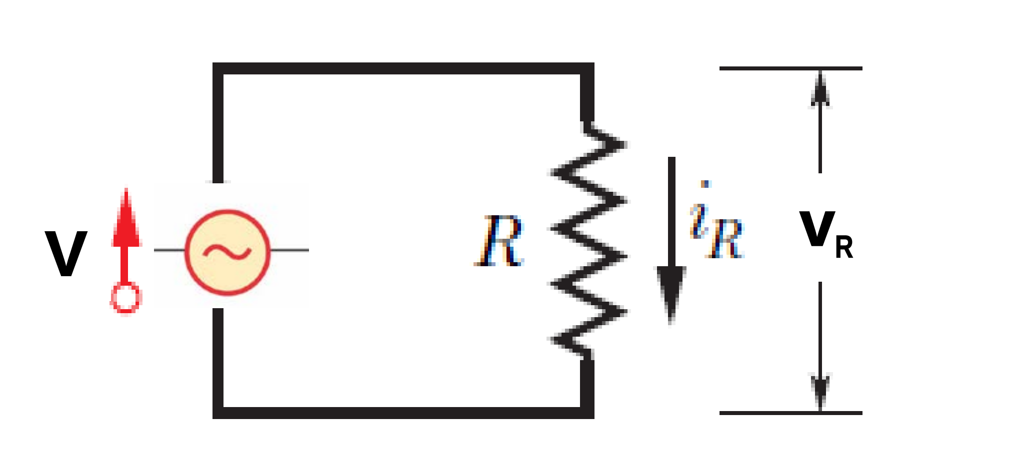
❖ Work, Energy and Power

- Work – energy spent to overcome a restraint to achieve a physical change
- Energy is the ability to do work
- Energy and work have the same SI units – Joules (J)
- Power – rate at which energy is expended (Joules/sec)
- Unit of power = **watt = Joules per second = Js^{-1}**
- Power (P) = Work (W)/Time (t); $W = Pt$
- When electrical current flows through a resistance, electrical energy is converted to heat at a rate that depends on the voltage across the resistance and the value of current through it:
  - Power (P) = Voltage (V) × Current (I) (watts)
  - $P = V^2/R = I^2R$ (watts)
- Power 'delivered' to a resistance = power dissipated in resistance
- A **kilowatt-hour** (**kWh**) is the total energy delivered or consumed in one hour, and is used in industry
- kWh = (P in kW) × (t in hours)

Electrical Circuit Fundamentals

- Circuit Theory-One
  - Ohms Law-One

- The greater the voltage at a source, the greater the current it can produce
- Current produced in a resistor is **directly proportional** to the voltage of the source
- Resistance reduces the flow of current
- Current is **inversely proportional** to resistance,
- For a fixed resistance *R*, the current *I* increases with an increase in the voltage *V* at the source
- This is **Ohm's Law**, which is a linear relationship
- $V = IR$; $I = V/R$; $R = V/I$



Electrical Circuit Fundamentals

- Circuit Theory-Two
  - Ohms Law-Two

Ohm's law states that the current through a conductor between two points is directly proportional to the voltage across the two points. :

Summary of voltage-current relationship		
Element	Time domain	Frequency domain
R	$v = Ri$	$V = RI$
L	$v = L\frac{di}{dt}$	$V = j\omega LI$
C	$i = C\frac{dv}{dt}$	$V = \frac{I}{j\omega C}$

Electrical Circuit Fundamentals

- Circuit Theory-Three
  - Ohms Law-Three

Impedances and admittances of passive elements		
Element	**Impedance**	**Admittance**
R	$Z = R$	$Y = \frac{1}{R}$
L	$Z = j\omega L$	$Y = \frac{1}{j\omega L}$
C	$Z = \frac{1}{j\omega C}$	$Y = j\omega C$

Electrical Circuit Fundamentals

- Circuit Theory-Four
  - Electric Circuit-One

An **electrical circuit or network** is an interconnection of electrical components (e.g., batteries, voltage sources, resistors, inductors, capacitors, switches, transistors) or a model of such an interconnection, consisting of electrical elements (e.g., voltage sources, current sources, resistances, inductances, capacitances). An **electrical circuit** is a network consisting of a closed loop, giving a return path for the current. Linear electrical networks, a special type consisting only of sources (voltage or current), linear lumped elements (resistors, capacitors, inductors), and linear distributed elements (transmission lines), have the property that signals are linearly superimposable. They are thus more easily analyzed, using powerful frequency domain methods such as Laplace transforms, to determine DC response, AC response, and transient response.

Electrical Circuit Fundamentals

- ## Circuit Theory-Five
  - ### Electric Circuit-Two

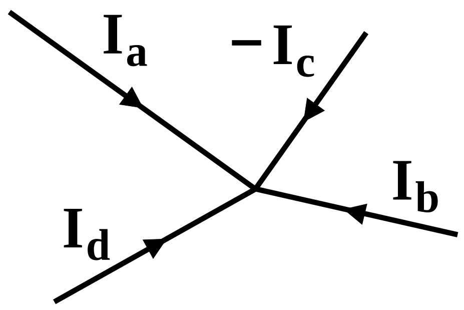
- An **electric circuit** is a configuration of interconnected resistors, active sources or other electrical components through which current flows
- A **complete circuit** is a circuit where there is a path from the source back to the source
- A **circuit diagram** is also known as a **schematic**
- **Circuit analysis** is the process of determining current flows and voltages that exist is various parts of a circuit.
- There are two basic kinds of connections that we will encounter when we analyse circuits
- 1) **Series**
- 2) **Parallel**

Electrical Circuit Fundamentals

❖ Circuit Theory-Six

❖ Kirchhoff's Current Law

- **The sum of all currents entering a junction, or any portion of a circuit, equals the sum of currents leaving the same**
- Kirchhoff's current law is a useful technique for problem solving, and it is often used to find unknown current values, as we shall see.



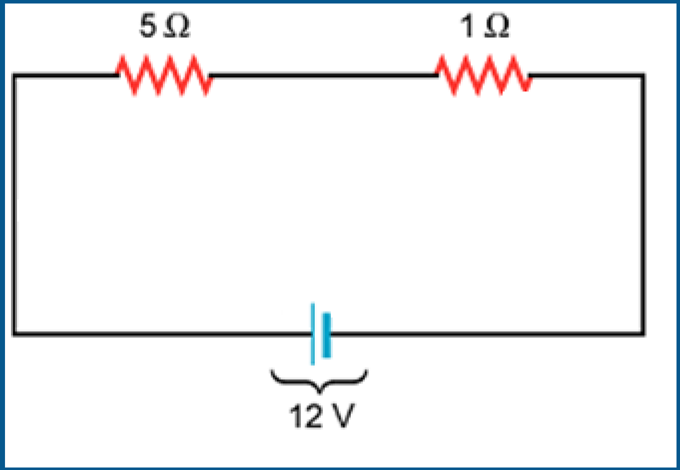
$$I_a + I_b - I_c + I_d = 0$$

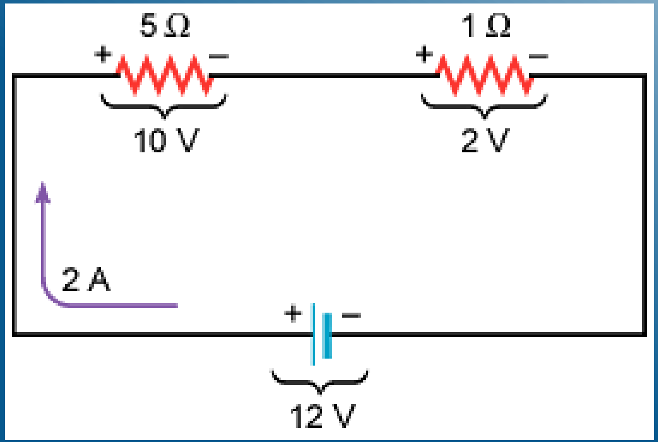
I_a, I_b, I_c, and I_d can each be either a positive or negative number.

Electrical Circuit Fundamentals

- Circuit Theory-Seven
  - Kirchhoff's Voltage Law

- **Kirchhoff's Voltage Law:** The sum of the voltage drops around any closed loop equals the sum of the voltage rises around the loop.





Loop rule. Around any closed circuit loop, the sum of the potential drops equals the sum of the potential rises.

Electrical Circuit Fundamentals

- Circuit Theory-Eight
  - Open Circuit

- An **open circuit** is a gap, break or interruption in a circuit path.
- Open circuits result in no current flow in a series circuit where there is a break.
- If the current I = 0, then from Ohm's Law $R = V/I$, which leads to $V/0 = \infty = R$
- Therefore at an open circuit, there is an infinite resistance
- We may think of an open circuit as a fault, but it can be useful for circuit analysis
- Loads are often measured in terms of its resistance, but we may need to study a circuit when its load is removed, i.e. when $R_L = \infty$
- **Important** $V \neq 0$ in an open circuit.
- From the above, $I = 0$ regardless of the value of V

Electrical Circuit Fundamentals

- AC Circuits-One
  - AC Circuits-One

In our analysis of the Texas Energy Problem we need to become familiar with Three Phase Electric Power Systems.

- An AC (Alternating Current) circuit consists of a combination of circuit elements and a power source.
- The power source provides an alternating voltage
- In an electrical transmission system the power source is a turbine connected to a synchronous generator
- The synchronous generator produces three phase alternating current and voltage.

Electrical Circuit Fundamentals

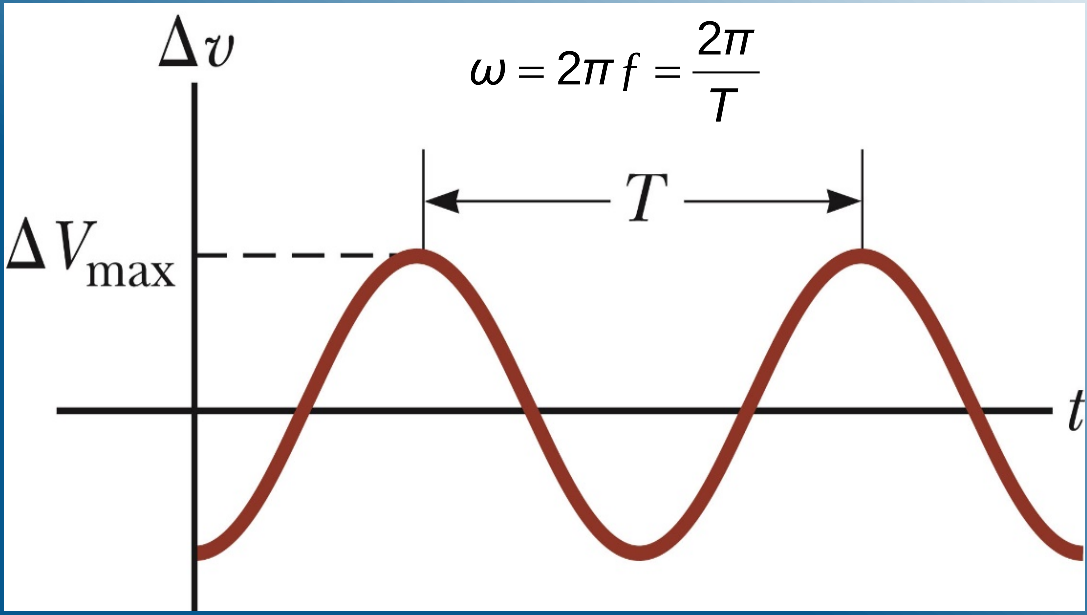
- AC Circuits-Two
  - AC Voltage-One

- The output of an AC power source is sinusoidal and varies with time according to the following equation:
  - $\Delta v = \Delta V_{max} \sin \omega t$
    - Δv is the instantaneous voltage.
    - ΔV_{max} is the maximum output voltage of the source.
      - Also called the **voltage amplitude**
    - ω is the angular frequency of the AC voltage.

Electrical Circuit Fundamentals

- AC Circuits-Three
  - AC Voltage-Two

- The angular frequency is
  - f is the frequency of the source.
  - T is the period of the source.
- The voltage is positive during one half of the cycle and negative during the other half.



Electrical Circuit Fundamentals

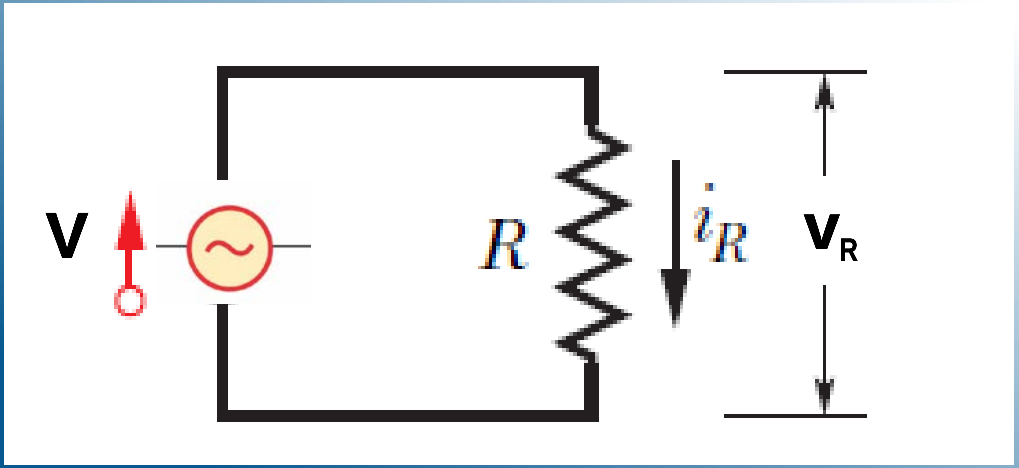
- AC Circuits-Four
  - AC Current

- The current in any circuit driven by an AC source is an alternating current that varies sinusoidally with time.
- Commercial electric power plants in the US use a frequency of 60 Hz.
  - This corresponds with an angular frequency of 377 rad/s.

Electrical Circuit Fundamentals

- AC Circuits-Five
  - Resistor in AC Circuit-One

- Consider a circuit consisting of an AC source and a resistor.
- The AC source is symbolized by 
- $\Delta v_R = \mathrm{D}V_{max} = V_{max}\ \sin \dot{\omega} t$
- Δv_R is the instantaneous voltage across the resistor.



- The instantaneous current in the resistor is

$$i_R = \frac{\Delta v_R}{R} = \frac{\Delta V_{max}}{R} \sin \omega t = I_{max} \sin \omega t$$

- The instantaneous voltage across the resistor is also given as

$$\Delta v_R = I_{max} R \sin \omega t$$

Electrical Circuit Fundamentals

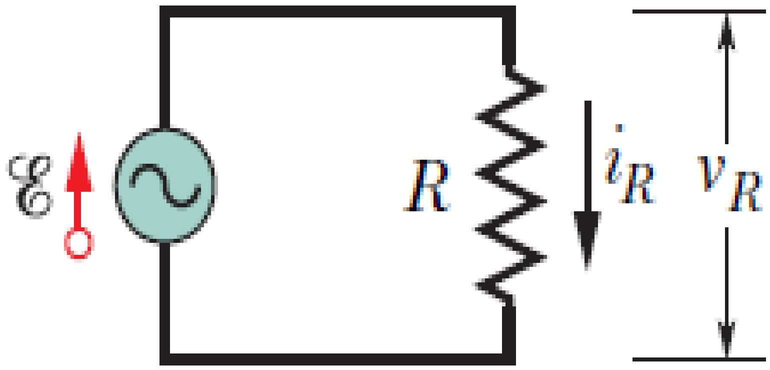
- AC Circuits-Six
  - Resistor in AC Circuit-Two

$$\mathscr{E} - v_R = 0.$$

$$v_R = \mathscr{E}_m \sin \omega_d t. = V_R \sin \omega_d t.$$

$$i_R = \frac{v_R}{R} = \frac{V_R}{R} \sin \omega_d t.$$

$$i_R = I_R \sin(\omega_d t - \phi),$$



A resistor is connected across an alternating-current generator

For a purely resistive load the phase constant is $\phi = 0°$

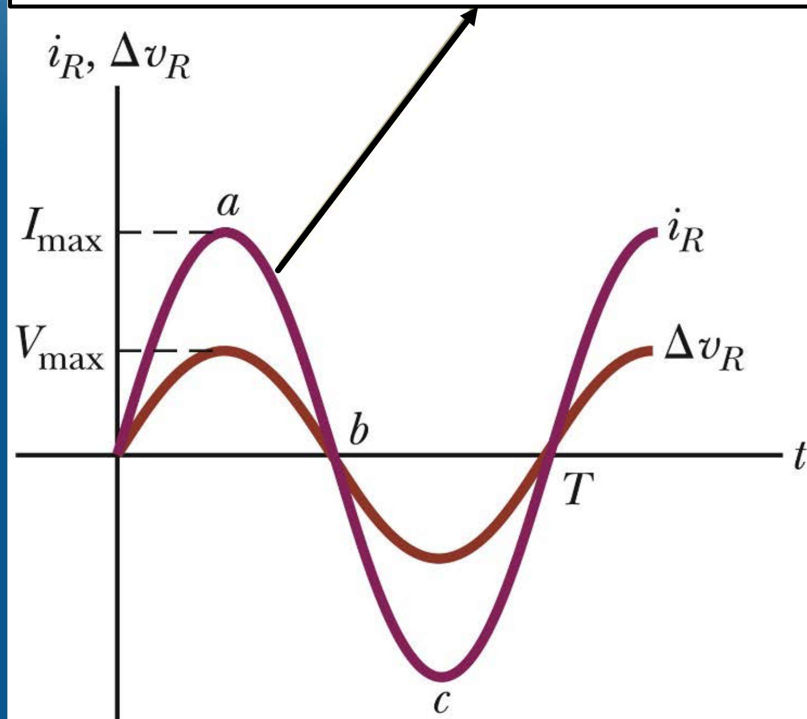
Electrical Circuit Fundamentals

❖ AC Circuits-Seven

❖ Resistor in AC Circuit-Three

- The graph shows the current through and the voltage across the resistor.
- The current and the voltage reach their maximum values at the same time.
- The current and the voltage are said to be *in phase.*
- For a sinusoidal applied voltage, the current in a resistor is always in phase with the voltage across the resistor.
- The direction of the current has no effect on the behavior of the resistor.
- Resistors behave essentially the same way in both DC and AC circuits.

The current and the voltage are in phase: they simultaneously reach their maximum value, their minimum values and their zero values.



Electrical Circuit Fundamentals

❖ Phasor diagrams: One

- An alternating voltage can be presented in different representations.
- One graphical representation is using rectangular coordinates.
  - The voltage is on the vertical axis.
  - Time is on the horizontal axis.
- The phase space in which the phasor is drawn is similar to polar coordinate graph paper.
  - The radial coordinate represents the amplitude of the voltage.
  - The angular coordinate is the phase angle.
  - The vertical axis coordinate of the tip of the phasor represents the instantaneous value of the voltage.
  - The horizontal coordinate does not represent anything.
- Alternating currents can also be represented by phasors.

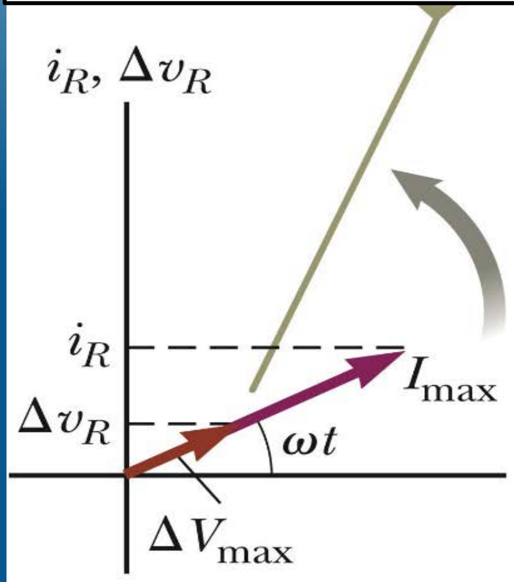
Electrical Circuit Fundamentals

❖ Phasor diagrams: Two

To simplify the analysis of AC circuits, a graphical constructor called a *phasor diagram* can be used.

- A **phasor** is a vector whose length is proportional to the maximum value of the variable it represents.
- The vector rotates counterclockwise at an angular speed equal to the angular frequency associated with the variable.
- The projection of the phasor onto the vertical axis represents the instantaneous value of the quantity it represents.

The current and the voltage are in the same direction because the current is in phase with the voltage.



The current and the voltage in the diagram above are plotted as current and voltage phasors.

Electrical Circuit Fundamentals

❖ RMS Current and Voltage: One

- The average current in one cycle is zero.
- Resistors experience a temperature increase which depends on the magnitude of the current, but not the direction of the current.
- The power is related to the square of the current.
- The rms current is the average of importance in an AC circuit.
    - *rms* stands for root mean square
- Alternating voltages can also be discussed in terms of rms values.

$$\Delta V_{rms} = \frac{\Delta V_{max}}{\sqrt{2}} = 0.707\,\Delta V_{max}$$

$$I_{rms} = \frac{I_{max}}{\sqrt{2}} = 0.707\,I_{max}$$

Electrical Circuit Fundamentals

❖ RMS Current and Voltage: Two

- rms values are used when discussing alternating currents and voltages because
  - AC ammeters and voltmeters are designed to read rms values.
  - Many of the equations that will be used have the same form as their DC counterparts.

Electrical Circuit Fundamentals

❖ Power as Energy Delivery

- The rate at which electrical energy is delivered to a resistor in the circuit is given by
  - $P = i^2 R$
    - i is the *instantaneous current.*
    - The heating effect produced by an AC current with a maximum value of I_{max} is not the same as that of a DC current of the same value.
    - The maximum current occurs for a small amount of time.
  - The average power delivered to a resistor that carries an alternating current is

$$P_{av} = I_{rms}^2 R$$

Electrical Circuit Fundamentals

❖ Inductors in AC Circuits: One

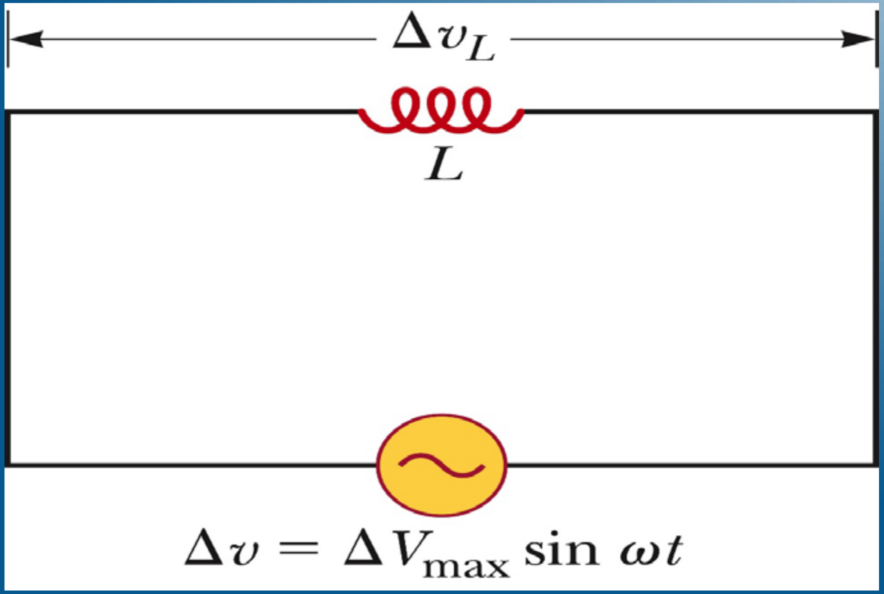
The most important reactive component in transmission theory is the inductor. High voltage transmission lines are almost always represented as lumped resistors and lumped inductors.

The inductor is a static device which stores magnetic energy. Using the drawing on the right and measuring the voltage as we go around the loop we get:

$$\Delta v + \Delta v_L = 0, \text{ or}$$

$$\Delta v - L\frac{di}{dt} = 0$$

$$\Delta v = L\frac{di}{dt} = \Delta V_{max}\sin\omega t$$



Electrical Circuit Fundamentals

❖ Inductors in AC Circuits: Two

- The factor ωL has the same units as resistance and is related to current and voltage in the same way as resistance.
- Because ωL depends on the frequency, it reacts differently, in terms of offering resistance to current, for different frequencies.
- The factor is the **inductive reactance** and is given by:
  - $X_L = \omega L$

Electrical Circuit Fundamentals

❖ Inductors in AC Circuits: Three

❖ Current in an Inductor

- The equation obtained from Kirchhoff's loop rule can be solved for the current

$$i_L = \frac{\Delta V_{\max}}{L}\int \sin \omega t \; dt = -\frac{\Delta V_{max}}{\omega L} \cos \omega t$$

$$i_L = \frac{\Delta V_{max}}{\omega L} sin\left(\omega t - \frac{\pi}{2}\right) \qquad I_{max} = \frac{\Delta V_{max}}{\omega L}$$

- This shows that the instantaneous current i_L in the inductor and the instantaneous voltage Δv_L across the inductor are out of phase by (p/2) rad = 90°.

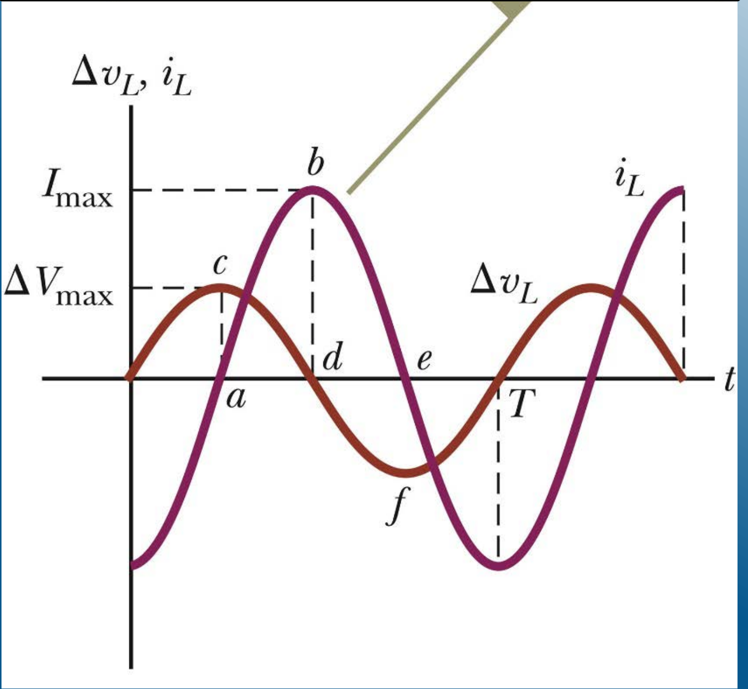
Electrical Circuit Fundamentals

❖ Inductors in AC Circuits: Four

❖ Phase Relationship of Inductor

- The current is a maximum when the voltage across the inductor is zero.
  - The current is momentarily not changing
- For a sinusoidal applied voltage, the current in an inductor always lags behind the voltage across the inductor by 90° ($\pi/2$).

The current lags the voltage by one-fourth of a cycle.



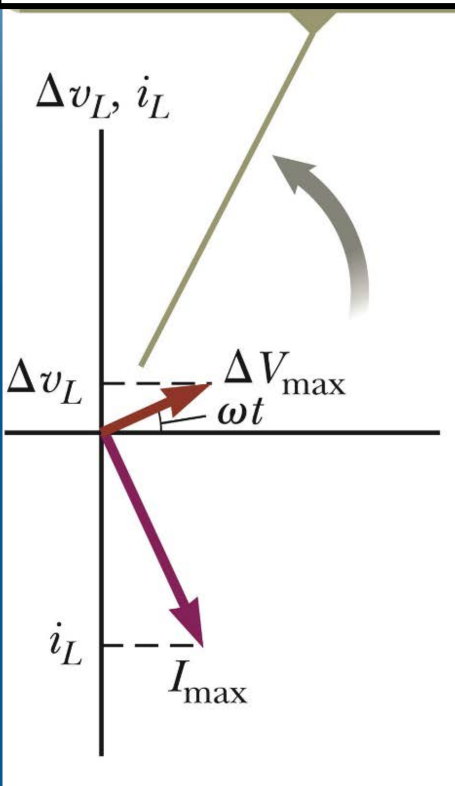
Electrical Circuit Fundamentals

❖ Inductors in AC Circuits: Five

❖ Phaser Diagram of Inductor

- The phasors are at 90º with respect to each other.
- This represents the phase difference between the current and voltage.
- Specifically, the current lags behind the voltage by 90º.

The current and voltage phasors are at 90º to each other.



Electrical Circuit Fundamentals

❖ Inductors in AC Circuits: Six

Current can be expressed in terms of the inductive reactance:

$$I_{\text{max}} = \frac{\Delta V_{\text{max}}}{X_L} \text{ or } I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{X_L}$$

As the frequency increases, the inductive reactance increases

This is consistent with Faraday's Law:

The larger the rate of change of the current in the inductor, the larger the back emf, giving an increase in the reactance and a decrease in the current.

Electrical Circuit Fundamentals

❖ Inductors in AC Circuits: Seven

The instantaneous voltage across the inductor is

$$\Delta v_L = -L\frac{di}{dt}$$

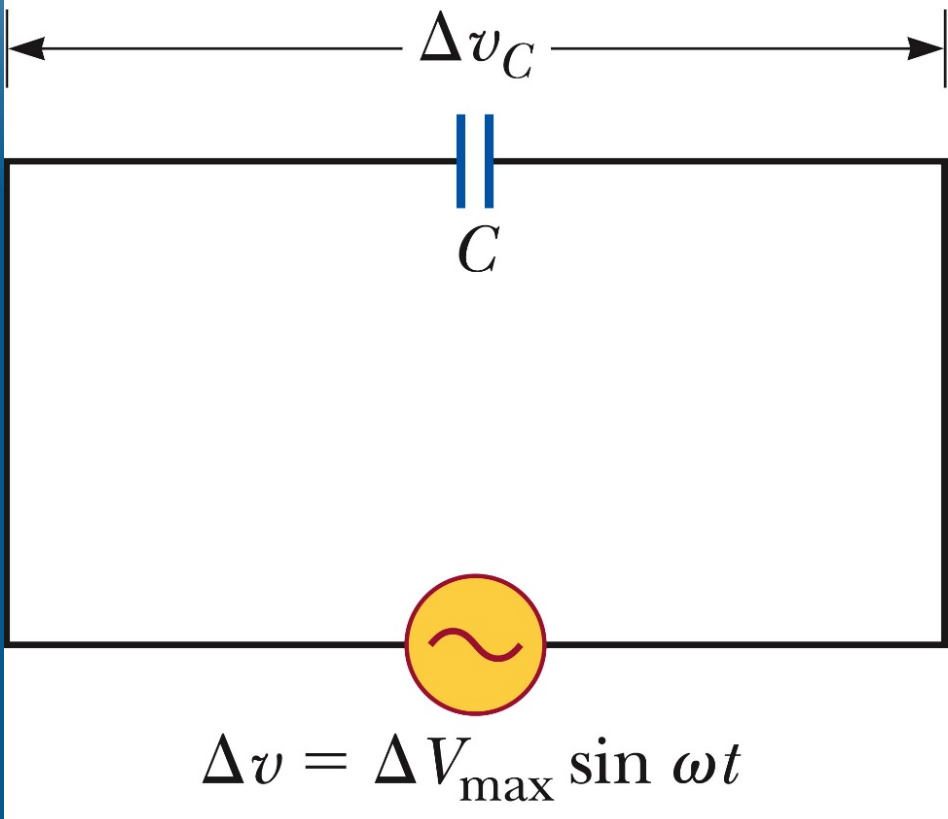
$$= -\Delta V_{\max} \sin \omega t$$

$$= -I_{\max} X_L \sin \omega t$$

Electrical Circuit Fundamentals

❖ Capacitors in AC Circuits: One

- The circuit contains a capacitor and an AC source.
- Kirchhoff's loop rule gives:
- $\Delta v + \Delta v_c = 0$ and so
- $\Delta v = \Delta v_C = \Delta V_{max} \sin \omega t$
  - Δv_c is the instantaneous voltage across the capacitor.



Electrical Circuit Fundamentals

❖ Capacitors in AC Circuits: Two

- The charge is $q = C\Delta V_{max} \sin \omega t$
- The instantaneous current is given by

$$i_C = \frac{dq}{dt} = \omega C \Delta V_{\max} \cos \omega t$$

$$\text{or } i_C = \omega C \Delta V_{\max} \sin\left(\omega t + \frac{\pi}{2}\right)$$

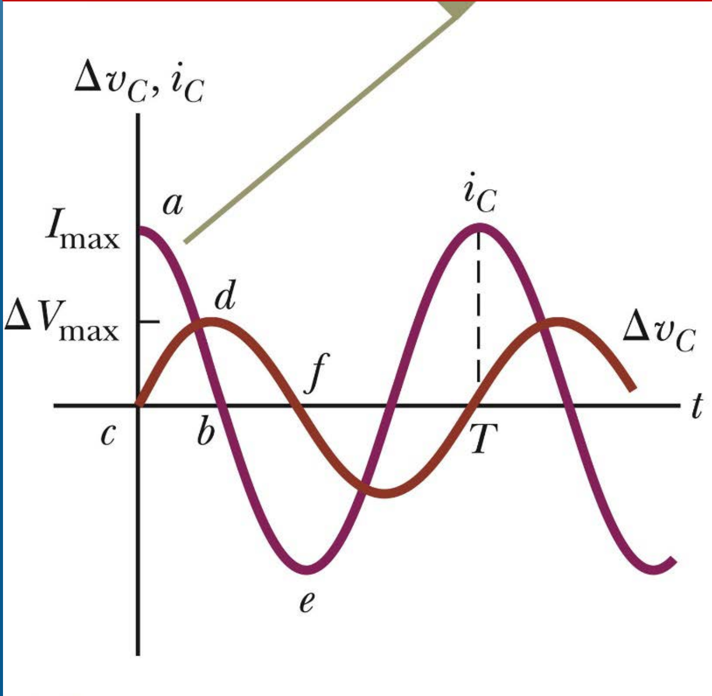
- The current is p/2 rad = 90º out of phase with the voltage

Electrical Circuit Fundamentals

❖ Capacitors in AC Circuits: Three

- ❖ The current reaches its maximum value one quarter of a cycle sooner than the voltage reaches its maximum value.
- ❖ The current leads the voltage by 90º.

The current leads the voltage by one-fourth of a cycle.



Electrical Circuit Fundamentals

❖ Capacitors in AC Circuits: Four

❖ Capacitive Reactance

- The maximum current in the circuit occurs at cos $\omega t = 1$ which gives

$$I_{\max} = \omega C \Delta V_{\max} = \frac{\Delta V_{\max}}{(1/\omega C)}$$

- The impeding effect of a capacitor on the current in an AC circuit is called the **capacitive reactance** and is given by

$$X_C \equiv \frac{1}{\omega C} \quad \text{which gives} \quad I_{\max} = \frac{\Delta V_{\max}}{X_C}$$

Electrical Circuit Fundamentals

❖ Capacitors in AC Circuits: Five

❖ Voltage Across Capacitor

- The instantaneous voltage across the capacitor can be written as $\Delta v_C = \Delta V_{max} \sin \omega t = I_{max} X_C \sin \omega t$.
- As the frequency of the voltage source increases, the capacitive reactance decreases and the maximum current increases.
- As the frequency approaches zero, X_C approaches infinity and the current approaches zero.
  - This would act like a DC voltage and the capacitor would act as an open circuit.

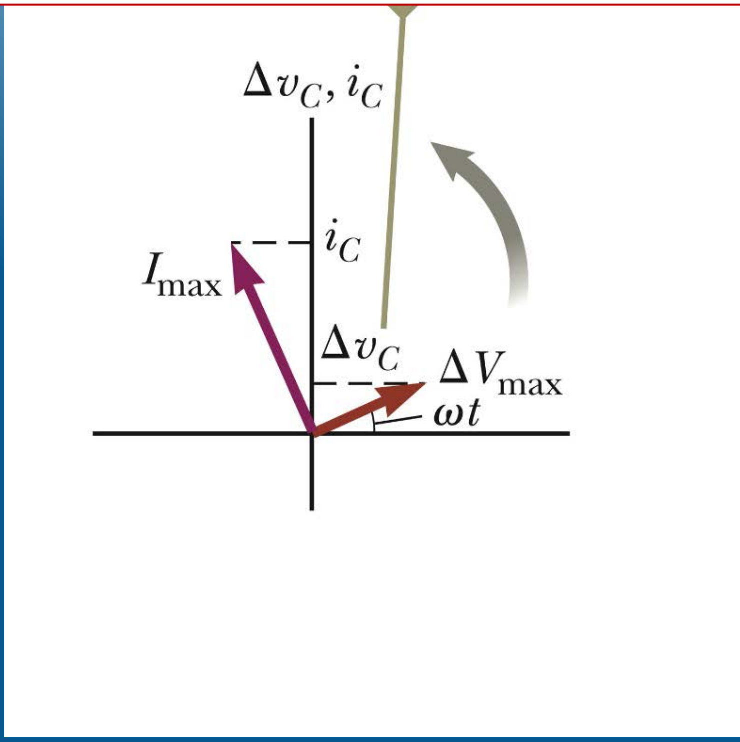
Electrical Circuit Fundamentals

❖ Capacitors in AC Circuits: Six

❖ Capacitor Phasor Diagram

The current and voltage phasors are 90º to each other.

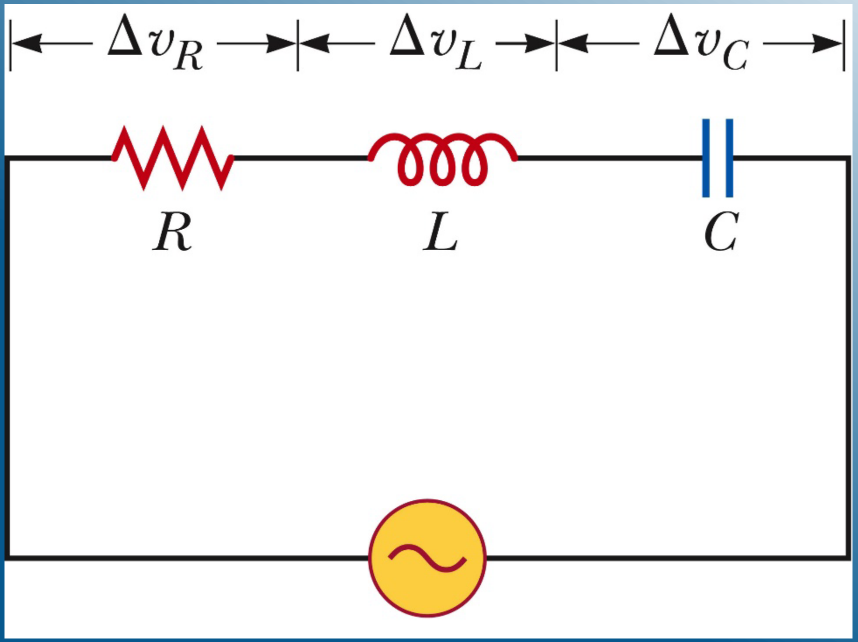
- ❖ The phasor diagram shows that for a sinusoidally applied voltage, the current always leads the voltage across a capacitor by 90º.



Electrical Circuit Fundamentals

❖ RLC SERIES Circuit-One

- ❖ The resistor, inductor, and capacitor can be combined in a circuit.
- ❖ The current and the voltage in the circuit vary sinusoidally with time.



Electrical Circuit Fundamentals

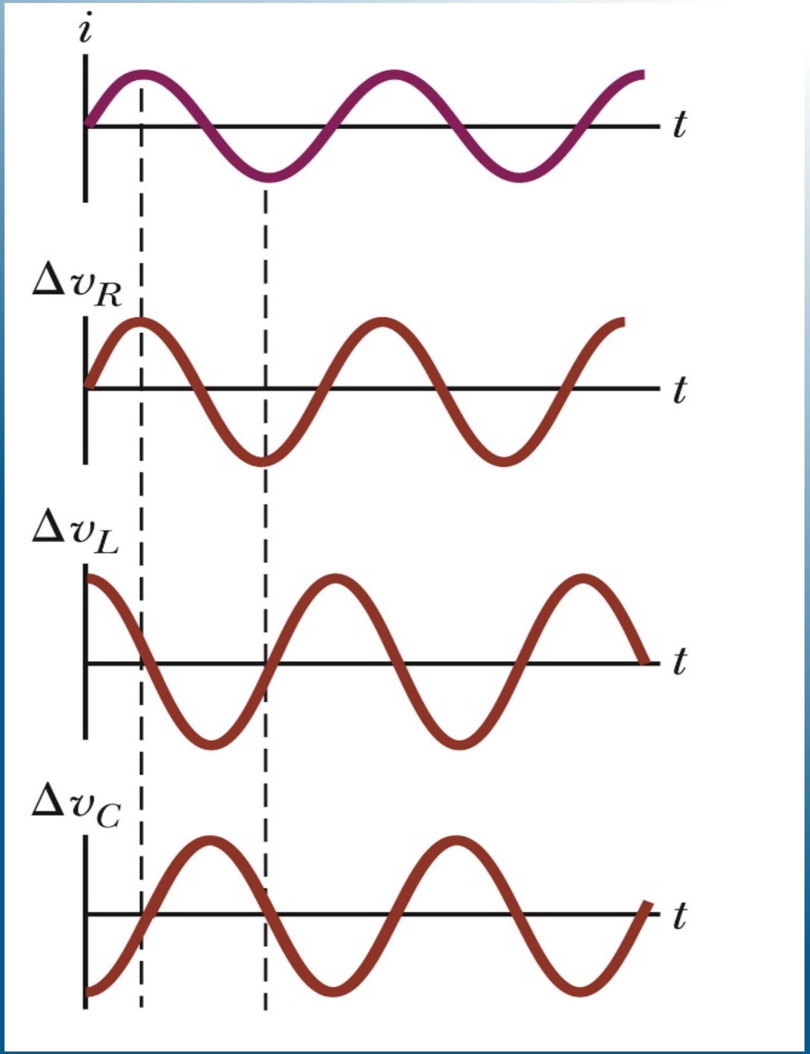
❖ RLC SERIES Circuit-Two

- The instantaneous voltage would be given by $\Delta v = \Delta V_{max} \sin \omega t$.
- The instantaneous current would be given by $i = I_{max} \sin(\omega t - \varphi)$.
  - φ is the **phase angle** between the current and the applied voltage.
- Since the elements are in series, the current at all points in the circuit has the same amplitude and phase.

Electrical Circuit Fundamentals

❖ RLC SERIES Circuit-Three

- The instantaneous voltage across the resistor is in phase with the current.
- The instantaneous voltage across the inductor leads the current by 90°.
- The instantaneous voltage across the capacitor lags the current by 90°.

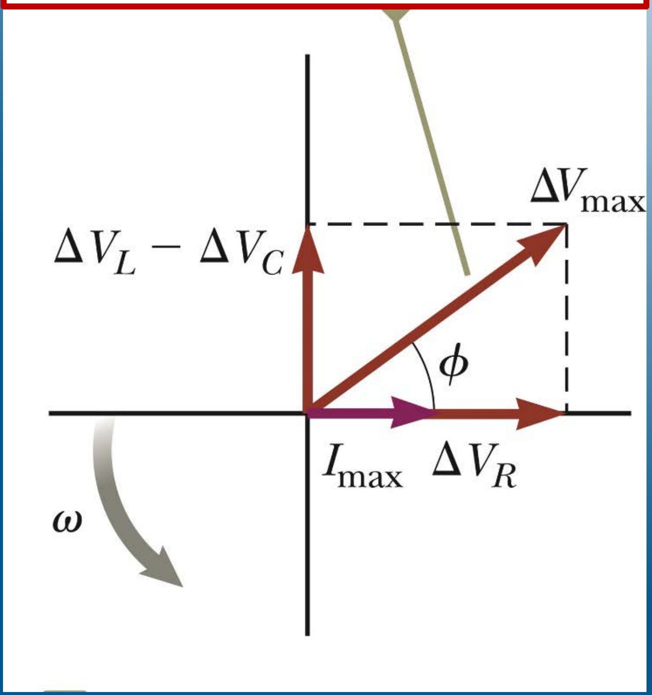


Electrical Circuit Fundamentals

❖ RLC SERIES Circuit-Four

- Vector addition is used to combine the voltage phasors.
- ΔV_L and ΔV_C are in opposite directions, so they can be combined.
- Their resultant is perpendicular to $\Delta V_{R.}$
- The resultant of all the individual voltages across the individual elements is $\Delta v_{max.}$
  - This resultant makes an angle of φ with the current phasor $I_{max.}$

The total voltage $\Delta v_{max.}$ Makes an angle of φ with the current phasor I_{max}



Electrical Circuit Fundamentals

❖ RLC SERIES Circuit-Five

The instantaneous voltage across each of the three circuit elements can be expressed as

$$\Delta v_R = I_{\max} R \sin \omega t = \Delta V_R \sin \omega t$$

$$\Delta v_L = I_{\max} X_L \sin\left(\omega t + \frac{\pi}{2}\right) = \Delta V_L \cos \omega t$$

$$\Delta v_C = I_{\max} X_C \sin\left(\omega t - \frac{\pi}{2}\right) = -\Delta V_C \cos \omega t$$

Electrical Circuit Fundamentals

❖ RLC SERIES Circuit-Six

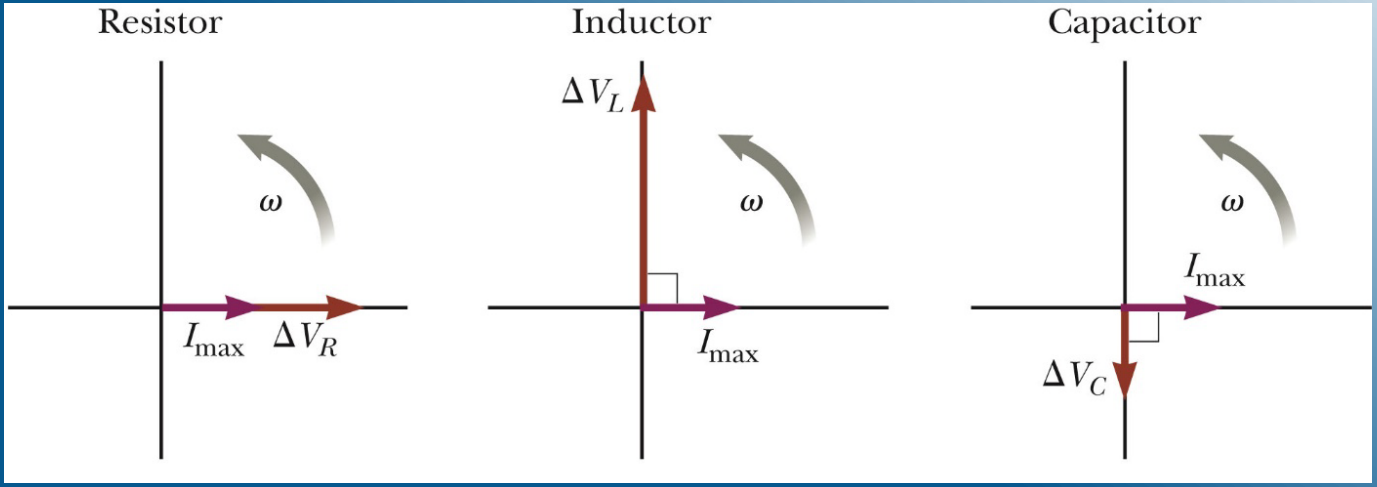
Characteristics of Voltage in RLC Circuits

- ΔV_R is the maximum voltage across the resistor and $\Delta V_R = I_{max}R$.
- ΔV_L is the maximum voltage across the inductor and $\Delta V_L = I_{max}X_L$.
- ΔV_C is the maximum voltage across the capacitor and $\Delta V_C = I_{max}X_C$.
- The sum of these voltages must equal the voltage from the AC source.
- Because of the different phase relationships with the current, they cannot be added directly.

Electrical Circuit Fundamentals

❖ RLC Series Circuit-Seven

❖ Phasor Diagram-One



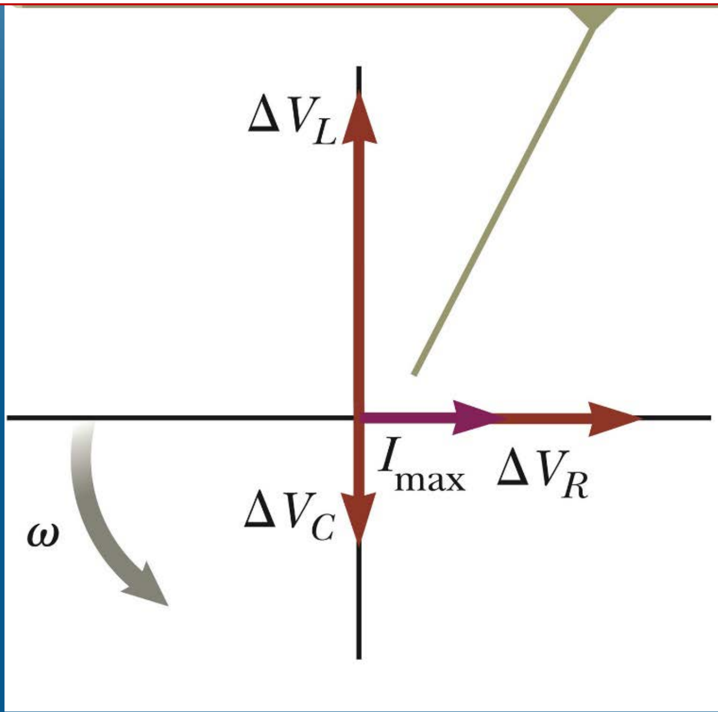
- To account for the different phases of the voltage drops, vector techniques are used.
- Remember the phasors are rotating vectors
- The phasors for the individual elements are shown.

Electrical Circuit Fundamentals

- RLC Series Circuit-Eight
  - Phasor Diagram-Two

- The individual phasor diagrams can be combined.
- Here a single phasor I_{max} is used to represent the current in each element.
  - In series, the current is the same in each element.

The current and voltage phasors are combined on a single set of axes.



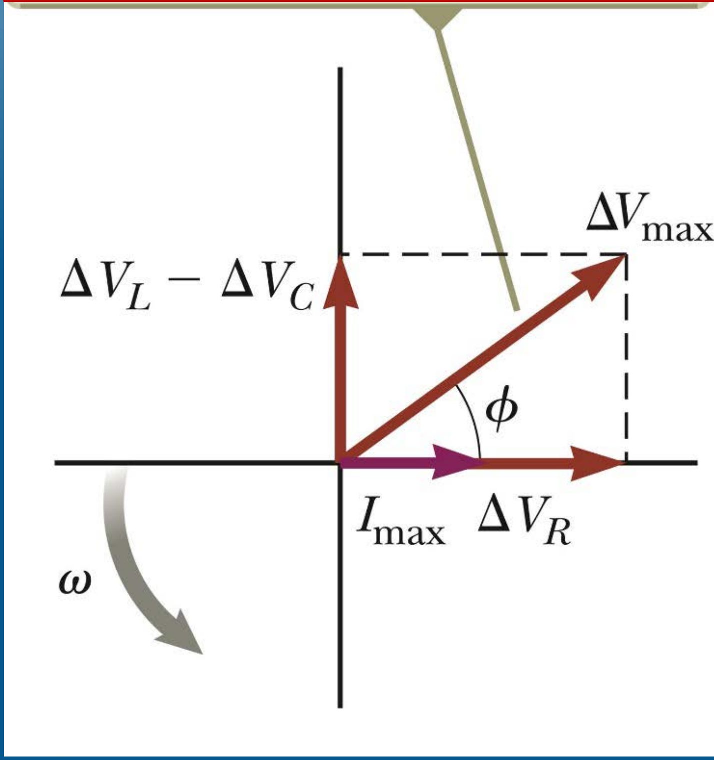
Electrical Circuit Fundamentals

RLC Series Circuit-Nine

Phasor Diagram-Three

- Vector addition is used to combine the voltage phasors.
- ΔV_L and ΔV_C are in opposite directions, so they can be combined.
- Their resultant is perpendicular to $\Delta V_{R.}$
- The resultant of all the individual voltages across the individual elements is $\Delta v_{max.}$
- This resultant makes an angle of φ with the current phasor $I_{max.}$

The total voltage Δv_{max} makes an angle of φ with I_{max}



Electrical Circuit Fundamentals

❖ RLC Series Circuit-Ten

❖ Total Voltage in RLC Circuits

- ❖ From the vector diagram, ΔV_{max} can be calculated

$$\Delta V_{\max} = \sqrt{\Delta V_R^2 + \left(\Delta V_L - \Delta V_C\right)^2}$$

$$= \sqrt{(I_{\max} R)^2 + \left(I_{\max} X_L - I_{\max} X_C\right)^2}$$

$$\Delta V_{\max} = I_{\max} \sqrt{R^2 + \left(X_L - X_C\right)^2}$$

Electrical Circuit Fundamentals

- RLC Series Circuit-Eleven
  - Impedance of RLC Circuit

- The current in an *RLC* circuit is

$$I_{\max} = \frac{\Delta V_{\max}}{\sqrt{R^2 + \left(X_L - X_C\right)^2}} = \frac{\Delta V_{\max}}{Z}$$

- Z is called the impedance of the circuit and it plays the role of resistance in the circuit, where

$$Z \equiv \sqrt{R^2 + \left(X_L - X_C\right)^2}$$

  - Impedance has units of ohms

Electrical Circuit Fundamentals

❖ RLC Series Circuit-Twelve

❖ Phase Angle of RLC Circuit

- ❖ The right triangle in the phasor diagram can be used to find the phase angle, φ.

$$\varphi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

- ❖ The phase angle can be positive or negative and determines the nature of the circuit.

Electrical Circuit Fundamentals

❖ RLC Series Circuit-Thirteen

❖ Sign of the Reactance of Circuit

- If f is positive
  - $X_L > X_C$ (which occurs at high frequencies)
  - The current lags the applied voltage.
  - The circuit is *more inductive than capacitive.*
- If f is negative
  - $X_L < X_C$ (which occurs at low frequencies)
  - The current leads the applied voltage.
  - The circuit is *more capacitive than inductive.*
- If f is zero
  - $X_L = X_C$
  - The circuit is *purely resistive.*

Electrical Circuit Fundamentals

❖ RLC Series Circuit-Fourteen

❖ Power in AC Circuit-One

- The average power delivered by the AC source is converted to internal energy in the resistor.
  - $P_{avg} = \frac{1}{2} I_{max} \Delta V_{max} \cos \varphi = I_{rms} \Delta V_{rms} \cos \varphi$
  - $\cos \varphi$ is called the power factor of the circuit
- We can also find the average power in terms of R.
  - $P_{avg} = I^2_{rms} R$
- When the load is purely resistive, $\varphi = 0$ and $\cos \varphi = 1$
  - $P_{avg} = I_{rms} \Delta V_{rms}$

Electrical Circuit Fundamentals

❖ RLC Series Circuit-Fifteen

❖ Power in AC Circuit-Two

- The average power delivered by the source is converted to internal energy in the resistor.
- No power losses are associated with pure capacitors and pure inductors in an AC circuit.
  - In a capacitor, during one-half of a cycle, energy is stored and during the other half the energy is returned to the circuit and no power losses occur in the capacitor.
  - In an inductor, the source does work against the back emf of the inductor and energy is stored in the inductor, but when the current begins to decrease in the circuit, the energy is returned to the circuit.
- The power delivered by an AC circuit depends on the phase.
- Some applications include using capacitors to shift the phase to heavy motors or other inductive loads so that excessively high voltages are not needed.

Electrical Circuit Fundamentals

❖ RLC Series Circuit-Sixteen

❖ Resonance in AC Circuit-One

- *Resonance* occurs at the frequency ω_o where the current has its maximum value.
  - To achieve maximum current, the impedance must have a minimum value.
  - This occurs when $X_L = X_C$
  - Solving for the frequency gives

$$\omega_o = \frac{1}{\sqrt{LC}}$$

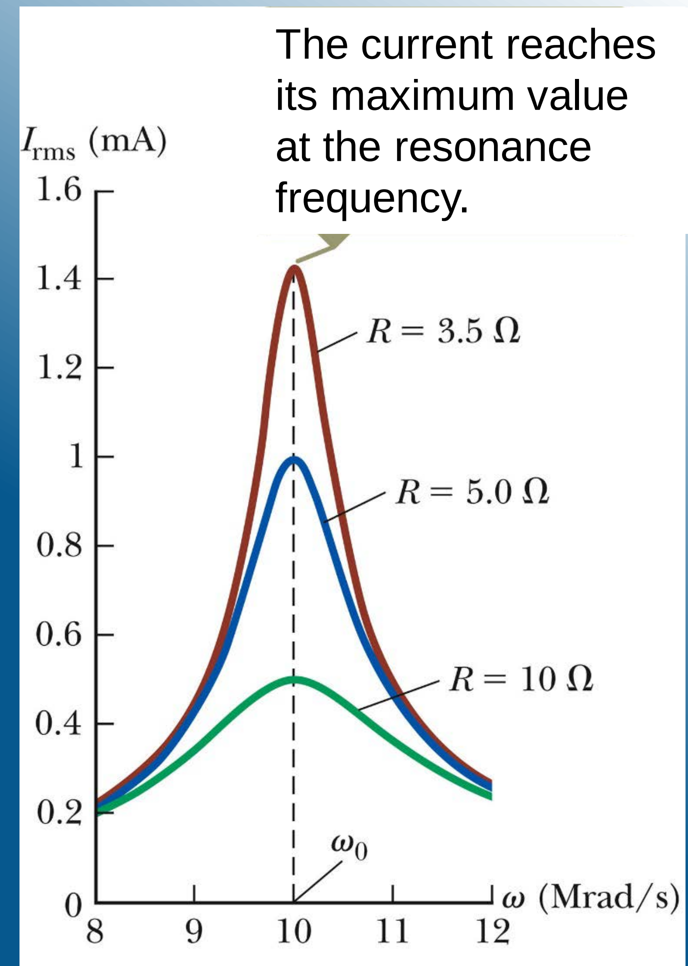
- The resonance frequency also corresponds to the natural frequency of oscillation of an *LC* circuit.
- The rms current has a maximum value when the frequency of the applied voltage matches the natural oscillator frequency.
- At the resonance frequency, the current is in phase with the applied voltage.

Electrical Circuit Fundamentals

❖ RLC Series Circuit-Seventeen

❖ Resonance in AC Circuit-Two

- ❖ Resonance occurs at the same frequency regardless of the value of *R.*
- ❖ As *R* decreases, the curve becomes narrower and taller.
- ❖ Theoretically, if *R* = 0 the current would be infinite at resonance.
  - ❖ Real circuits always have some resistance.



Electrical Circuit Fundamentals

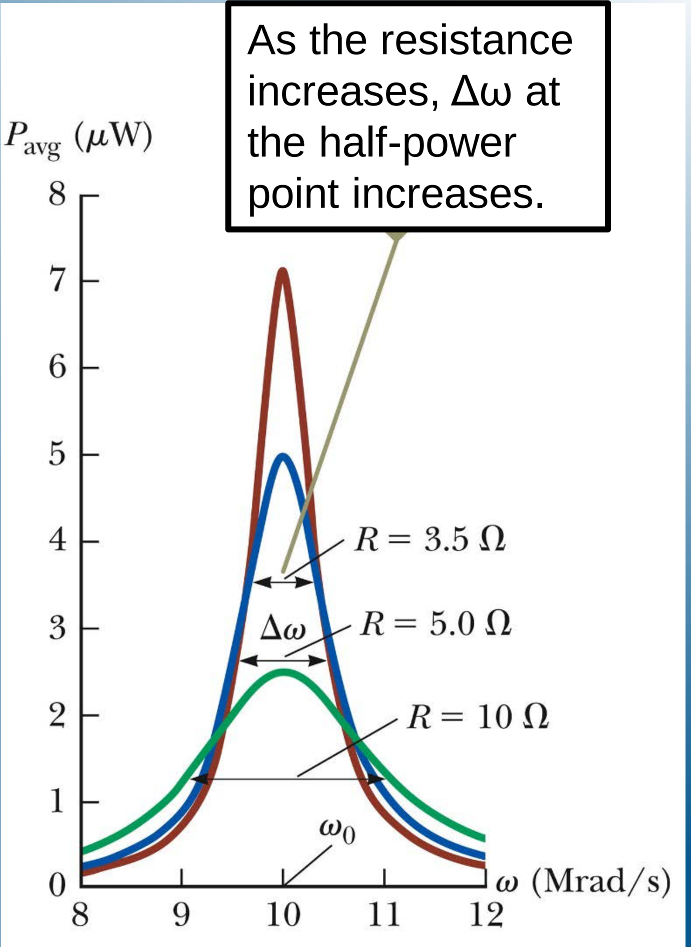
❖ RLC Series Circuit-Eighteen

❖ Power as a function of frequency

- ❖ Power can be expressed as a function of frequency in an *RLC* circuit.

$$P_{av} = \frac{\left(\Delta V_{rms}\right)^2 R\omega^2}{R^2\omega^2 + L^2\left(\omega^2 - \omega_o^2\right)^2}$$

- ❖ This shows that at resonance, the average power is a maximum.



Electrical Circuit Fundamentals

❖ RLC Series Circuit-Eighteen

❖ Quality Factor

- The sharpness of the resonance curve is usually described by a dimensionless parameter known as the quality factor, Q.
- $Q = \omega_o / \Delta\omega = (\omega_o L) / R$
  - $\Delta\omega$ is the width of the curve, measured between the two values of ω for which P_{avg} has half its maximum value.
    - These points are called the *half-power points.*
- A high-*Q* circuit responds only to a narrow range of frequencies.
  - Narrow peak
- A low-*Q* circuit can detect a much broader range of frequencies.
- A radio's receiving circuit is an important application of a resonant circuit.